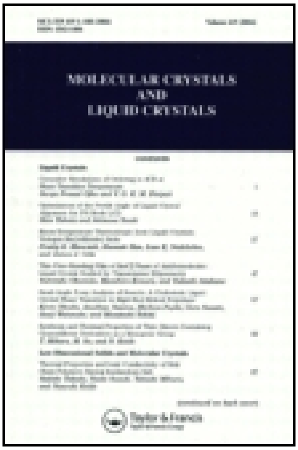

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Edge Modes in Chiral Liquid Crystals: Options for Low Threshold Lasing

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Edge Modes in Chiral Liquid Crystals: Options for Low Threshold Lasing

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An analytic theory of the localized edge modes (EM) in chiral liquid crystals (CLC) is developed. Equations determining the edge mode frequencies are found and analytically solved for the case of low decaying modes and solved numerically for the problem parameters values typical for the experiment. The discrete EM frequencies specified by the integer numbers n are located close to the stop band edge frequencies outside the band. The expressions for space distribution of the n's mode field in CLC layer and for its temporal decay are presented. The possibilities of reduction of the lasing threshold due to the anomalously strong absorption effect are theoretically investigated for a distributed feedback lasing in CLC. It is shown that a minimum of the threshold pumping wave intensity may be reached, generally, for the pumping wave propagating at an angle to the helical axes. However, for lucky values of the related parameters it may be reached for the pumping wave propagating along the helical axis. The lowest threshold pumping wave intensity occurs for the lasing at the first low frequency band-edge lasing mode and the pumping wave propagating at an angle to the spiral axes corresponding to the first absorption maximum of the anomalously strong absorption effect at the high frequency edge of stop band. The corresponding analytical study is performed for the case of the average dielectric constant of LC coinciding with the dielectric constant of the material limiting LC. Numerical calculations of the DFB lasing threshold at EM frequencies are performed for the typical values of the related parameters.

Keywords: anomalous absorption; chiral LC; low threshold lasing; stop band edge modes

INTRODUCTION

Recently there was explosion of interest to the mirrorless distributed feedback (DFB) lasing in chiral liquid crystals (CLC) [1]. The reason

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for this interest is connected with the observed low threshold lasing [2,3], unusual polarization properties of lasing and frequency tunability of the lasing by means of applying external field [4], temperature pitch variations [5,6], or by application of a mechanical stress [7] etc.

The DFB low threshold lasing in CLC occurs at frequencies close to the frequencies of the stop band edges [2–7]. The corresponding frequencies were associated with, so called "edge lasing modes" [1]. It happens also that at the same edge lasing mode frequencies an anomalously strong absorption of the pumping wave occurs [8–11].

In general, the theory of the edge lasing modes in CLC (and more general DFB lasing in CLC) is very similar to the corresponding theory for conventional periodic solid media which initially was developed by Kogelnik [12] in the coupled wave approximation and later was treated by the analogous way in many papers [13]. However the theory of edge lasing modes in CLC deserves a separate study because of unusual optical properties of the chiral liquid crystals and because the fact that, unlike to all other periodic media, for chiral liquid crystals (and more general spiral media) an exact analytic solution of the Maxwell equations is known. So many related results, usually obtained in a numerical approach, may be obtained analytically for CLC. For example, the anomalously strong absorption effect existing in chiral liquid crystals [8,9] for the light frequency close to the stop band may be treated analytically (The anomalously strong absorption effect for conventional periodic media was studied also in [14]).

Below general analytic expressions for the solution of the boundary problem for the nonabsorbing, absorbing and amplifying CLC, i.e., for the "edge modes" (EM), are presented for the light propagation direction coinciding with the spiral axes. The dispersion equation for the edge lasing modes determining their frequencies and the lasing threshold gain is found and an expression for the threshold in a specific limiting case is presented as well as numerical solutions of the dispersion equation for typical values of the CLC parameters are found. The properties of the EM (coordinate intensity distribution, frequency width of the EM etc.) are analyzed. It is discussed also how the revealed properties of EMs allow to decrease the DFB lasing threshold ensured as by a low gain for the lasing so by a strong absorption for the pumping wave.

EIGEN WAVES IN CHIRAL LC

To solve the boundary problem related to EM one needs to know eigenwaves in CLC. As it is known [9,15,16] the eigenwaves corresponding to propagation of light in chiral LC along a spiral axes, i.e., the solution of the Maxwell equation

$$\partial^2 E/\partial z^2 = c^{-2}\varepsilon(z)\partial^2 E/\partial t^2, \tag{1}$$

are presented by a superposition of two plane waves of the form

$$E(z,t) = e^{-i\omega t}[E^+ n_+ \exp(iK^+ z) + E^- n_- \exp(iK^- z)] \tag{2}$$

where ω is the light frequency, $n_\pm$ are the two vectors of circular polarizations, $\varepsilon(z)$ is the dielectric tensor of the chiral liquid crystal [9,15,16], c is the light velocity and the wave vectors $K^\pm$ satisfy to the condition

$$K^+ - K^- = \tau, \tag{3}$$

where τ is the reciprocal lattice vector of the LC spiral ($\tau = 4\pi/p$, where p is the cholesteric pitch).

The wave vectors $K^\pm$ in four eigen solutions of (1) are determined by the Eq. (3) and the following formulas

$$K_j^+ = \tau/2 \pm \kappa\{1 + (\tau/2\kappa)^2 \pm [(\tau/\kappa)^2 + \delta^2]^{1/2}\}^{1/2}, \tag{4}$$

where j numerates the eigen solutions with the ratio of amplitudes (E^-/E^+) given by the expression

$$(E^-/E^+)_j = \delta/[(K_j^+ - \tau)^2/\kappa^2 - 1], \tag{5}$$

where $\kappa = \omega\varepsilon_0^{1/2}/c$, $\varepsilon_0 = (\varepsilon_\| + \varepsilon_\perp)/2$, $\delta = (\varepsilon_\| - \varepsilon_\perp)/(\varepsilon_\| + \varepsilon_\perp)$ is the dielectric anisotropy, and $\varepsilon_\|$, $\varepsilon_\perp$ are the principal values of the LC dielectric tensor [9,16,17]. Note that we do not specify what kind of chiral liquid crystals is under the investigation here, chiral smectics or cholesterics, because the optics of light propagating along the spiral axes is identical for the both type of these LC [9,16,17]. For the certainty we shall give below the expressions for cholesterics. The corresponding expressions for chiral smectics may be obtained by a simple redefinition of the related parameters (see [9], Chapt. 2).

Two of the eigen waves corresponding to the circular polarization with the sense of chirality coinciding with the one of the CLC spiral experience strong diffraction scattering at the frequencies in the region of the stop band. Other two eigenwaves corresponding to the opposite circular polarizations are almost not influenced by the diffraction scattering even at the frequencies of the stop band for the former circular polarization.

Because, as we shall see, the specific of EM in CLC is connected with the eigen waves of diffracting polarization we shall limit ourselves below by consideration of propagation in CLC of light of the diffracting polarization only.

BOUNDARY PROBLEM

To investigate EM in CLC we have to consider a boundary problem. We shall assume that the CLC is presented by a planar layer with a spiral axes perpendicular to the layer surfaces (Fig. 1). To justify our intention to limit ourselves by consideration of propagation of light of the diffracting polarization only we shall assume also that the average dielectric constant of the CLC ε_0 coincides with the dielectric constant of the external medium. This assumption practically prevents conversion of one circular polarization to another one at the layer surfaces [9,18] and allows one to take into account in the consideration only two eigen waves with diffracting circular polarization.

Begin the consideration of linear boundary problem in the formulation which assumes that two plane wave of the diffracting polarization and of the same frequency are incident along the spiral axes at the layer from the opposite sides (see Fig. 1) and the dielectric tensor may have non zero imaginary part of any sign. The admission of any sign of the imaginary part of dielectric tensor means that the CLC layer may be as well absorbing so amplifying. The amplitudes of the two diffracting eigenwaves E_j^+ excited in the layer by the incident waves (they are denoted by E_+^+, E_-^+) are determined by the following

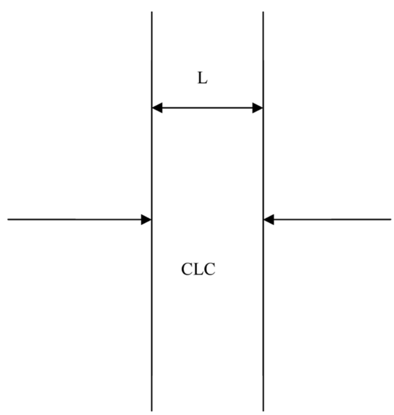


FIGURE 1 Schematic of the boundary problem for edge mode.

equations

$$\begin{aligned}
&E_+^+ + E_-^+ = E_{ir}\\
&\exp\left[iK_+^+L\right]\left\{\delta\Big/\left[\left(K_+^+-\tau\right)^2\Big/\kappa^2-1\right]\right\}E_+^+ +\\
&\exp\left[iK_-^+L\right]\left\{\delta\Big/\left[\left(K_-^+-\tau\right)^2\Big/\kappa^2-1\right]\right\}E_-^+ = E_{il}
\end{aligned}\tag{6}$$

where E_{ir} and E_{il} are the amplitudes of the waves incident at the layer from right and left side, respectively, L is the layer thickness and

$$K_\pm^+ = \tau/2 \pm \kappa\left\{1+(\tau/2\kappa)^2-\left[(\tau/\kappa)^2+\delta^2\right]^{1/2}\right\}^{1/2}. \tag{7}$$

The amplitudes of the waves exiting the layer from right E_{er} and left E_{el} side, respectively, are determined by the expressions

$$\begin{aligned}
&E_{er} = \left\{\delta\Big/\left[\left(K_+^+-\tau\right)^2\Big/\kappa^2-1\right]\right\}E_+^+ + \left\{\delta\Big/\left[\left(K_-^+-\tau\right)^2\Big/\kappa^2-1\right]\right\}E_-^+\\
&E_{el} = \exp\left[i\left(K_+^+-\kappa\right)L\right]E_+^+ + \exp\left[i\left(K_-^+-\kappa\right)L\right]E_-^+.
\end{aligned}\tag{8}$$

If one assumes that the amplitude only of one incident wave is not zero the Eq. (8) determines the reflected and transmitted waves (reflection R and transmission T coefficients of the layer) and, in particular, their frequency dependence [9,16,18]. The corresponding expressions for R and T take the form

$$\begin{aligned}
&R = \delta^2|\mathrm{sin}qL|^2\Big/\left|\left(q\tau/\kappa^2\right)\mathrm{cos}qL + i\left[(\tau/2\kappa)^2+(q/\kappa)^2-1\right]\mathrm{sin}qL\right|^2\\
&T = |\exp[i\kappa L](q\tau/\kappa^2)|^2/|(q\tau/\kappa^2)\mathrm{cos}qL\\
&\qquad + i\left[(\tau/2\kappa)^2+(q/\kappa)^2-1\right]\mathrm{sin}qL|^2,
\end{aligned}\tag{9}$$

where

$$q = \kappa\left\{1+(\tau/2\kappa)^2-\left[(\tau/\kappa)^2+\delta^2\right]^{1/2}\right\}^{1/2}. \tag{10}$$

If the both amplitude of the incident waves are equal to zero no waves emerging from the layer exist if the dielectric tensor has a positive (or a very small negative) imaginary part.

The solution of the system (6) determining the amplitudes E_+^+, E_-^+ of the eigenwaves in the CLC layer is given by the following expressions (for the case of a wave incident only at one surface of the layer):

$$
\begin{aligned}
E_+^+ &= -E_{il}\exp[-iqL]\left[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1 - q\tau/\kappa^2\right]\Big/2 \\
&\quad \left\{(q\tau/\kappa^2)\cos qL + i\left[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1\right]\sin qL\right\} \\
E_-^+ &= E_{il}\exp[iqL]\left[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1 + q\tau/\kappa^2\right]\Big/2 \\
&\quad \left\{(q\tau/\kappa^2)\cos qL + i\left[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1\right]\sin qL\right\}
\end{aligned}
\tag{11}
$$

The values of the eigenwave amplitudes close to the stop band edges are strongly oscillating functions of the frequency (see Figs. 2, 3 presenting the calculation results). At the points of maxima close to the stop band edges their values are much larger than the amplitude of incident wave E_{il}. It happens that the amplitude maxima frequencies just coincide with the frequencies of zero reflection following from (9) for nonabsorbing CLC (see Figs. 2, 3).

NONABSORBING LC, EDGE MODES

Let us examine in more details the formulas of the preceding section for nonabsorbing CLC. It means that $\gamma = 0$ in a general expression for the dielectric constant $\varepsilon = \varepsilon_0(1 + i\gamma)$ (Note, that at real situations $|\gamma| \ll 1$). The calculations of reflection R and transmission T coefficients as a function of the frequency for this case according Eqs. (9) (Figs. 2a, 3a) give the well known results [15–18]. Namely, a strong reflection inside the stop band and frequency oscillations of T and R outside of the stop band edges with $0 \le R \le 1$ and conserving of the relationship $T + R = 1$ for all frequencies.

The corresponding calculations of the amplitudes E_+^+, E_-^+ of the excited in the layer eigen waves (Figs. 2, 3b,c) reveal a nontrivial frequency dependence of E_+^+, E_-^+. Namely, close to the stop band edges (outside of the stop band edges) their frequency oscillations are accompanied by an essential enhancement of the amplitude values relative to the incident wave amplitude (in the calculations the incident wave amplitude assumed to be equal to 1). The thicker is a layer the higher is the enhancement (compare Figs. 2, 3). As the Figures 2, 3 show the positions of the amplitude oscillations maxima just coincide (or are very close) with the positions of reflection coefficient minima for nonabsorbing CLC at which $R = 0$.

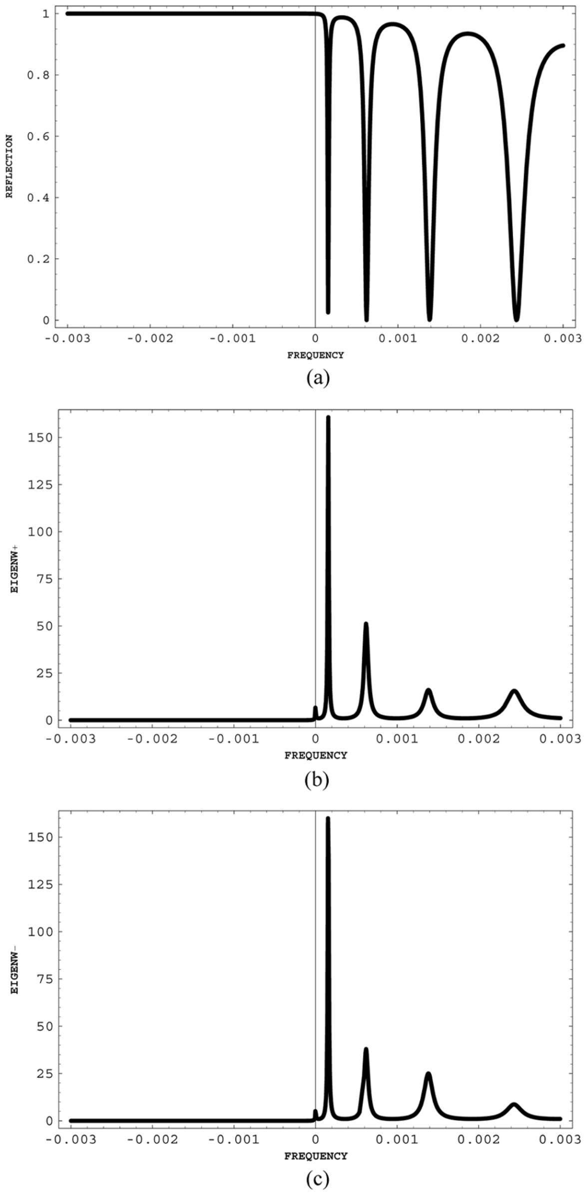


FIGURE 2 Reflection coefficient R (a), squared E_{+}^{+} (b) and E_{-}^{+} (c) eigen mode amplitudes calculated versus the frequency for nonabsorbing CLC layer ($\delta = 0.05$, $N = L/p = 250$). Here and at all figures below $\nu - 1$ is plotted at the frequency axes, i.e., is plotted the frequency deviation from the stop band edge (normalized by the Bragg frequency multiplied by δ, see Eqs. (19, 20)).

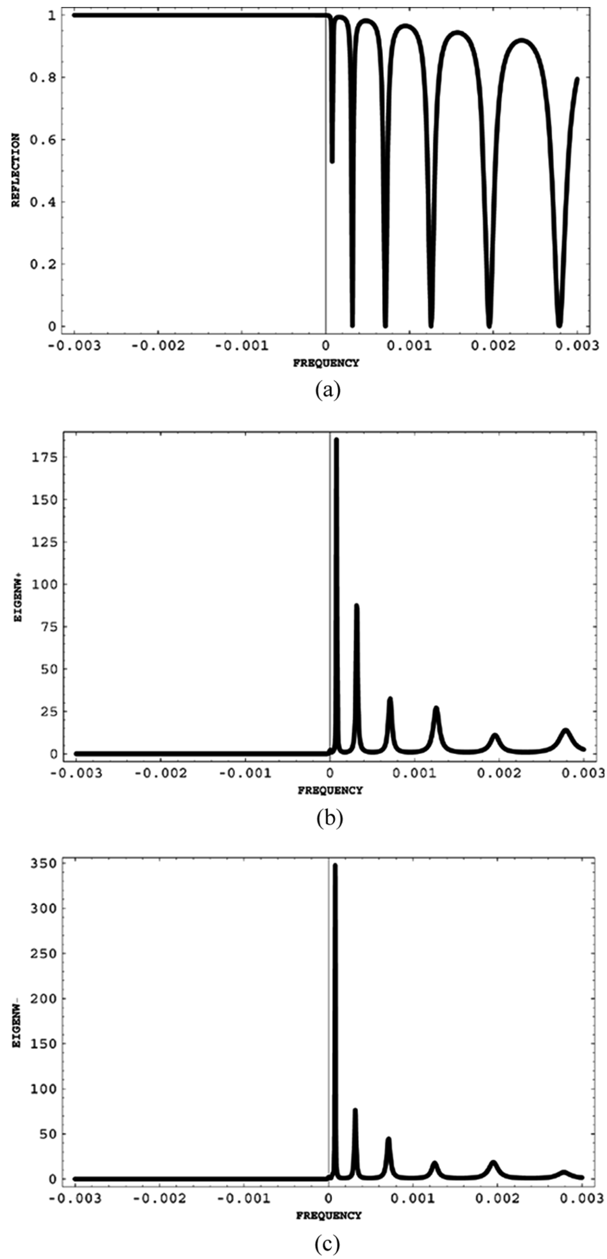


FIGURE 3 Reflection coefficient R (a), squared E_{+}^{+} (b) and E_{-}^{+} (c) eigen mode amplitudes calculated versus the frequency for nonabsorbing CLC layer ($\delta = 0.05$, $N = 350$).

The mentioned relationship between the amplitudes of eigen and incident waves at the specific frequencies shows that the energy of radiation in CLC at the layer thickness for these frequencies is much higher than the corresponding energy of the incident wave at the same thickness. So, in complete accordance with [1] one may to conclude that at the corresponding frequencies the incident wave excites some localized mode in CLC. To find this localized mode one has to solve the homogeneous system (6), i.e., the Eq. (6) with zero values of E_{ir} and E_{il}. The condition of solvability of the obtained homogeneous system determines the discrete frequencies of these localized modes:

$$\mathrm{tgqL} = i\left(q\tau/\kappa^2\right)\Big/\left[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1\right] \tag{12}$$

In a general case the solution of Eq. (12) determining the EM frequencies ω_{EM} may to be found only numerically. The EM frequencies ω_{EM} occur to be complex quantities which may be presented as $\omega_{EM} = \omega^0_{EM}(1 + i\Delta)$, where Δ in real situations is a small parameter. So, the localized modes are weakly decaying in time, i.e., they are quasistationary modes. Luckily, an analytic solution may be found for some limiting case, namely, for a sufficiently small Δ ensuring the condition $\mathrm{LImq} \ll 1$. In this case the ω^0_{EM} values are coinciding with the frequencies of zero values of reflection coefficient R for nonabsorbing CLC determined by the condition

$$qL = n\pi \quad \text{and} \quad \Delta = -\frac{1}{2}\delta(n\pi)^2/(\delta L\pi/4)^3, \tag{13}$$

where n is the edge mode number growing with departure of the frequency from the stop band edge (with $n = 1$ corresponding to the frequency closest to the stop band edge).

In the found solution of the homogeneous system (6) the eigen solutions amplitude ratio is $E^+_-/E^+_+ = -1$ and the field distribution inside the CLC layer is a superposition of two eigen waves given by (2) with the found amplitude ratio. The following from (2) explicit expression for the EM field distribution inside the CLC layer is given by the formula:

$$\begin{aligned} E(\omega_{EM}, z, t) = {} & i\exp(-i\omega_{EM}t)\{n_+\exp(i\tau z/2)\sin qz - n_-\exp(-i\tau z/2) \\ & \times \{[(\tau/\kappa)^2 + \delta^2]^{1/2}\sin qz + i[\tau q/(\delta\kappa)^2]\cos qz\}\}, \end{aligned} \tag{14}$$

where ω_{EM} is the EM frequency, and q is determined by Eq. (10).

For the mentoned above analytic solution the Eq. (14) for the EM field distribution inside the CLC layer accepts the form:

$$\begin{aligned}E(\omega_n, z, t) = i\exp(-i\omega_n t)\{&n_+ \exp(i\tau z/2)\sin(n\pi z/L)\\ &- n_- \exp(-i\tau z/2)\{[(\tau/\kappa)^2 + \delta^2]^{1/2}\sin(n\tau z/L)\\ &+ i[(\tau n\pi/L)/(\delta\kappa)^2]\cos(n\pi z/L)\}\},\end{aligned} \qquad (15)$$

The field distributions following from Eqs. (14, 15) for the EM numbers n = 1, 2, 3 are presented at Figure 4. The Figure 4 shows that the EM field is localized inside the CLC layer and its energy density experiences oscillations inside the layer with the number of the oscillations just equal to the EM number n.

The Figure 4 presents a total energy distribution in the layer. However, as it is clear from the Eqs. (2, 14, 15), in each point of the CLC layer the total field is presented by two plane wave directed in the opposite directions, so one may calculate separately for any point in the layer the intensities of the waves propagated in the opposite directions. In general, the coordinate distribution of the intensities of wave propagating in the opposite directions is similar to the one presented at Figure 4. However these distributions are of a special interest close to the layer surfaces. The Figure 5 shows the intensity coordinate distributions of the waves directed inside and outside of the layer close to the layer surfaces. One can see that at the layer surface the intensity

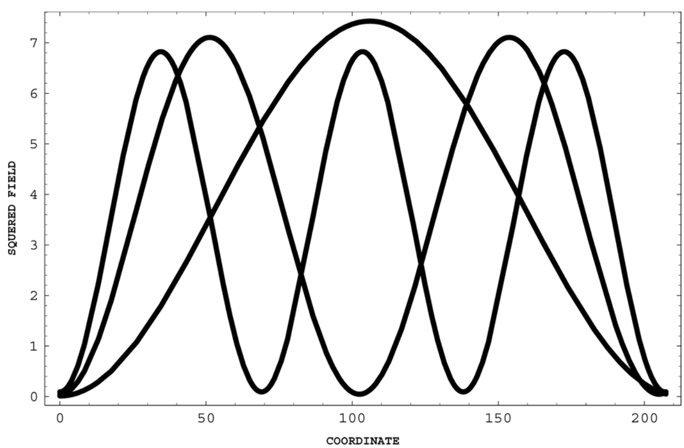


FIGURE 4 Calculated EM coordinate (in the dimensionless units $z\tau$) energy (arbitrary units) distribution inside the CLC layer for the three first edge modes ($\delta = 0.05$, N = 16.5, n = 1, 2, 3).

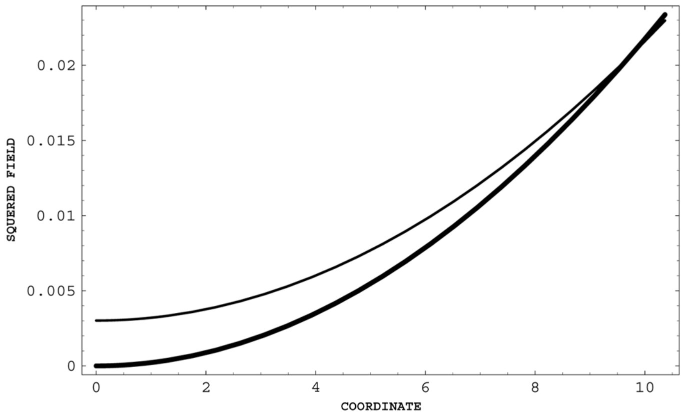


FIGURE 5 Calculated EM coordinate (in the dimensionless units $z\tau$) energy (arbitrary units) distribution close to the CLC layer surface for the plane wave directed inside (bold line) and outside the layer for the first edge mode ($\delta = 0.05$, $N = 16.5$, $n = 1$).

of the wave directed inside the layer is strictly zero, but for the same point the intensity of the wave directed outside the layer is not zero, however small. It means that the EM energy is leaking from the layer through its surfaces. From Eq. (15) follows the expression for the leaking wave amplitude at the CLC layer surface

$$E_{out} = (\tau n\pi/L)/(\delta \cdot \kappa)^2 \approx np/L\delta, \tag{16}$$

where p is the CLC pitch. The Eq. (16) shows that the energy leakage of EM is inversely proportional to the squared layer thickness L and proportional to the squared number n of EM. So, the most long living is the first EM in CLC layer. If $L\delta/p \gg 1$ the leaking wave amplitude E_{out}, as the Eq. (16) shows, is small, (i.e., $E_{out} < 1$).

Because for a nonabsorbing CLC layer (which is under consideration in this section) the only source of decay is the energy leakage through its surfaces, i.e., the decreasing of the EM energy in unite time is equal simply the energy flow of the leaking waves $(2c/\varepsilon_0^{1/2})$ $|E_{out}|^2$, one using (14–16) easily gets the following expression for the EM life-time τ_m

$$\begin{aligned}\tau_m &= \int (E(\omega_{EM}, z, t)|^2 dz \Big/ \left\{ d[\int |E(\omega_{EM}, z, t)|^2 dz|]/dt \right\} \\ &= [L\varepsilon_0^{1/2}/8c][1 + (\tau/\kappa)^2 + \delta^2 + (\tau q)^2/(\delta\kappa)^4]/[(\tau q)^2/\delta\kappa)^4]. \end{aligned} \tag{17}$$

At the condition LImq $\ll$ 1 the Eq. (17) is simplified

$$\begin{aligned}\tau_m &= [L\varepsilon_0^{1/2}/4c][1+(\tau/\kappa)^2+\delta^2+(np/L\delta^2)^2]/(np/L\delta^2)^2 \\ &\approx [5L\varepsilon_0^{1/2}/8c](L\delta^2/np)^2 \sim L^3/n^2. \end{aligned} \tag{18}$$

So, for sufficiently thick CLC layers for growing their thickness L the EM life-time τ_m growths as a third power of the thickness and is inversely proportional to the square of the EM number n. Note, that the same dependence of the life-time τ_m on n and L follows from Eq. (13).

The performed in this section analysis of the localized EM (solution of the homogeneous system following from Eq. (6)) together with the found in the preceding section solution of the inhomogeneous Eq. (6) allows one to discuss the ways and efficiency of the EM excitation. To excite EM in a nonamplifying CLC layer one need to have an external wave (waves) of the frequency coinciding with the EM frequency incident at the CLC layer. The general solution of the boundary problem found via the system (6) in this case may be presented as a superposition of the particular solution corresponding to the inhomogeneous system (6) and the solution corresponding to the homogeneous system (6), i.e., corresponding to EM, with the coefficient which has to be determined from the boundary conditions. One can easily construct such presentation of the boundary problem solution determined by Eq. (11). It happens that if only one plane wave (with unite amplitude) is incident at the CLC layer the amplitude of the excited EM it is given by the expression for E_+^+ in Eq. (11). However, in this case is impossible to excite alone EM only. It is accompanied by the E_-^+ eigen mode of approximately unite amplitude. The efficiency of the EM excitation by one plane wave (the ratio of the squared EM amplitude to the squared incident wave amplitude) for the specific values of related parameters is presented by the squared E_+^+ value at Figure 2b, 3b. It is possible to excite in the CLC layer EM only if two plane waves are incident at the layer from opposite sides and their phase difference is adjusted according to the structure of plane waves composing EM.

ABSORBING LC

Let us examine now EM in absorbing CLC. The motivation of this study, in particular, is the DFB lasing in CLC. One has to keep in mind that at lasing the CLC is an essentially absorbing medium for the pumping wave. Examine in more details the formulas of the preceding sections keeping in the mind their application to the pumping wave. Assume for simplicity that the absorption in LC is isotropic. Define the ratio of the dielectric constant imaginary part to the real part of ε as γ, i.e., $\varepsilon = \varepsilon_0(1 + i\gamma)$. Note, that at actual situations $\gamma \ll 1$. At Figures 6–8 the R, T, and 1-R-T calculated versus the frequency are presented for several values of positive and negative γ including its values close to the threshold values for the edge lasing modes (see below Eq. (22)). Due to the assumed isotropy of the absorption the frequency dependencies of the calculated characteristics are symmetric relative to the Bragg frequency (the mid point of the stop band), it is why only the frequencies above the Bragg frequency are presented at the figures. It is reasonable to comment here on the numerical values of the parameters used in the calculations. It is accepted that the dielectric anisotropy $\delta = 0.05$ what corresponds to a typical value of this parameter. The same may be said about the layer thickness L. Because for the cholesteric $\tau = 4\pi/p$, where p is the cholesteric pitch, the number of pitches N at the layer thickness L is equal to $l/4\pi$ $(l = L\tau)$ so the accepted in the calculations $l = 300$ corresponds to N close to 30, i.e., to a very common for the experiment number. All mentioned quantities reveal frequency beats close to the frequency edge of the selective reflection band. The positions of corresponding maxima and minima are determined by the layer thickness L, δ and are slightly dependent on the value of γ. In absorbing LC the sum of intensities of the reflected and transmitted beams is less than the intensity of the incident beam, i.e., $R + T < 1$. The equality holds only for nonabsorbing LC, i.e., in this case $R + T = 1$. As an example the positions of the beats minima of the reflection coefficient R (following from (9)) are given at Figure 2a for nonabsorbing LC, i.e., for $\gamma = 0$ which correspond to

$$\begin{gathered} qL = \pi n, \quad \pm\nu = 1 + (\pi n/a)^2/2, \quad n = 1, 2, 3, \ldots.., \\ \nu = 2(\omega - \omega_B)/\delta\omega_B, \quad \omega_B = c\tau/2\varepsilon_0^{1/2}, \quad a = \delta L\pi/4. \end{gathered} \tag{19}$$

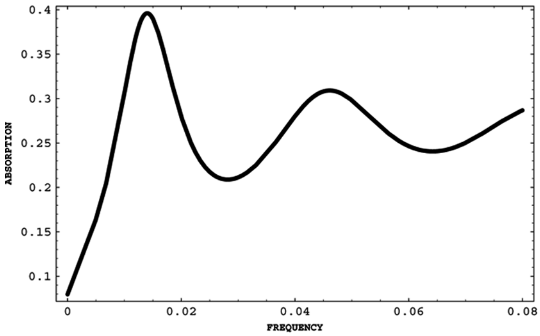
In a typical situation $a \gg 1$.

The edges of selective reflection band ω_e are connected to the Bragg frequency ω_B by the following relation

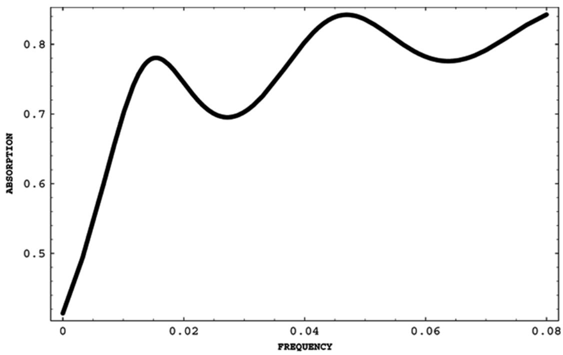
$$\omega_e = \omega_B/(1 \pm \delta)^{1/2} = c\tau/2[\varepsilon_0(1 \pm \delta)]^{1/2}. \tag{20}$$

So, at the edges $\nu = 2(1/(1 \pm \delta)^{1/2} - 1)/\delta \approx -(\pm 1)$.

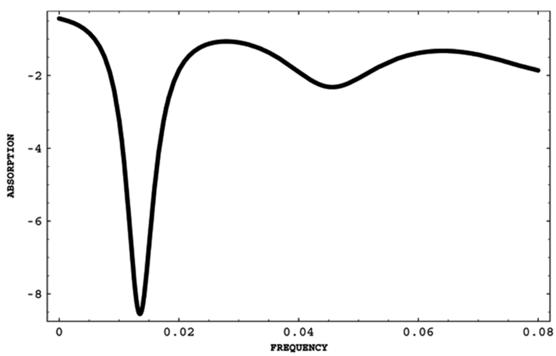
For small γ and $L\,\mathrm{Im}q \ll 1$ the reflection and transmission coefficients (9) at the frequencies of reflection minima (19) are reduced to



(a)



(b)



(c)

FIGURE 6 Absorption (1-R-T) calculated versus the frequency ($l = 300$, $l = L\tau = 4\pi N$) (a) for $\gamma = 0.001$, (b) $\gamma = 0.005$ (c) for a low amplifying layer (below the threshold gain for the first lasing edge mode, $\gamma = -0.003$); R (d) and T (e) calculated versus the frequency ($l = 300$, $l = L\tau$) for a low amplifying layer (below the threshold gain for the first lasing edge mode, $\gamma = -0.003$).

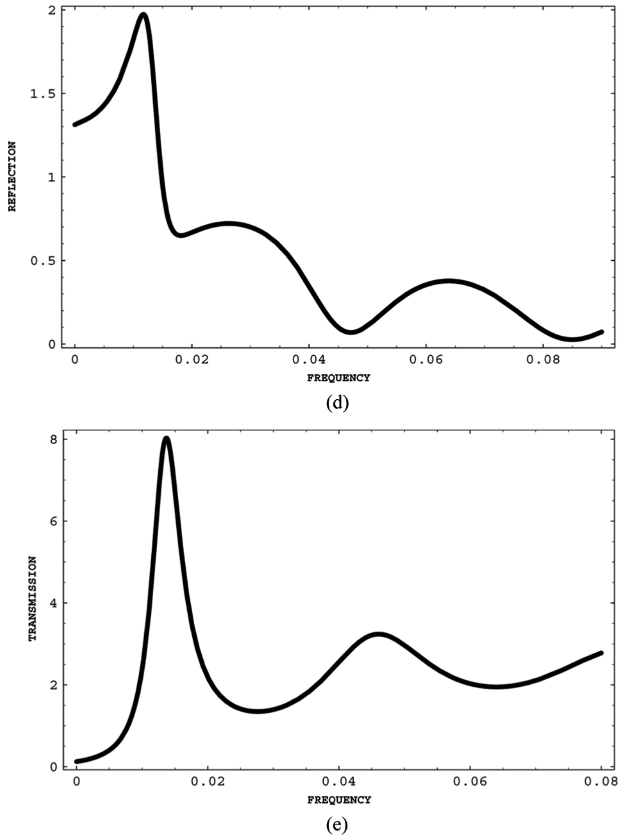


FIGURE 6 Continued.

the following expressions:

$$R = (a^3\gamma)^2/[(n\pi)^2 + a^3\gamma]^2, \qquad T = (n\pi)^4/[(n\pi)^2 + a^3\gamma]^2 \tag{21}$$

It follows from (19, 21) that for each n maximal absorption, i.e., maximal 1-R-T, occurs for $(n\pi)^2 = a^3\gamma$. It means that the maximal absorption occurs for a special relationship between δ, γ and L and if this relationship, i.e., $(n\pi)^2 = a^3\gamma$, is fulfilled $R = 1/4$, $T = 1/4$ and $1\text{-}R\text{-}T = 1/2$. Because the assumed smallness of γ this result corresponds to strong enhancement of the absorption for weakly absorbing layers.

As was shown in [8,9] just at the frequency values determined by (19) the effect of anomalously strong absorption reveals itself for

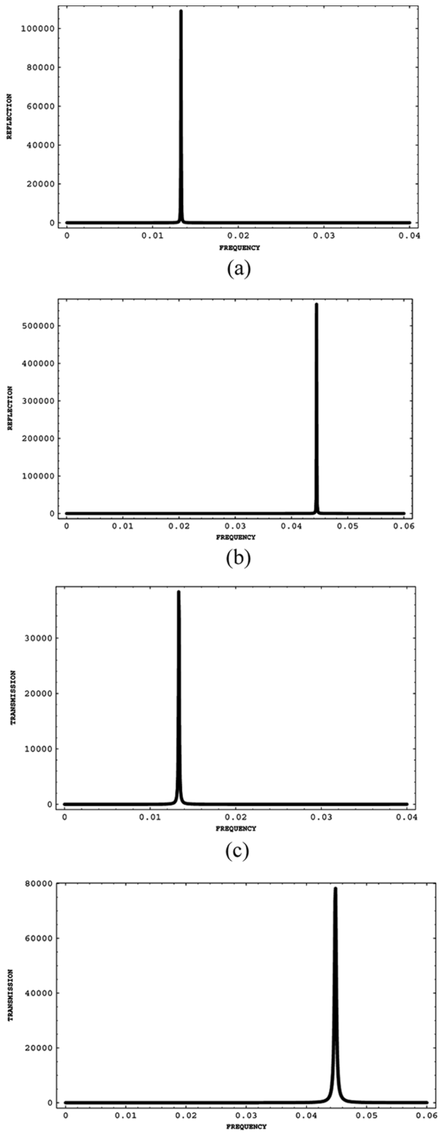


FIGURE 7 R calculated versus the frequency ($l = 300$, $l = L\tau$) (a) close to the threshold gain for the first lasing edge mode ($\gamma = -0.00565$), (b) close to the threshold gain for the second lasing edge mode ($\gamma = -0.0129$); T calculated versus the frequency ($l = 300$, $l = L\tau$) (c) close to the threshold gain for the first lasing edge mode ($\gamma = -0.00565$), (d) close to the threshold gain for the second lasing edge mode ($\gamma = -0.0129$).

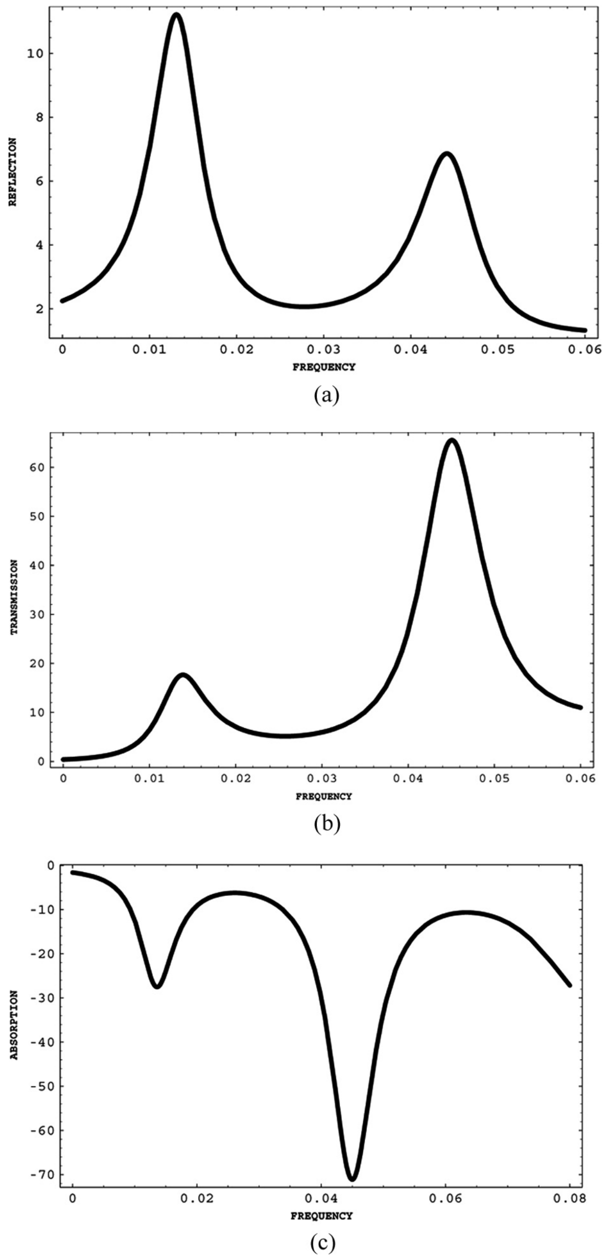


FIGURE 8 R (a), T (b), 1-R-T (c) calculated versus the frequency ($l = 300$, $l = L\tau$) for $\gamma = -0.009$, i.e., for the gain between the thresholds for the first and the second lasing edge modes.

absorbing chiral LC (Fig. 6a,b) and the edge modes for amplifying LC reveal themselves at lasing [1] (Fig. 7) (see similar results for layered media [12,13]). So, to minimize the intensity of the pumping wave which ensures lasing in chiral LC it is desirable to perform the pumping in conditions of the anomalously strong absorption effect and realization of lasing in the edge lasing mode. These options were investigated in details in [10] and will be discussed briefly in the following sections. In the conclusion of this section should be mentioned the following observation (compare Figs. 6–8): the absorption maxima in the frequency dependences are not so sharp as the intensity maxima for the lasing modes.

AMPLIFYING LC

Now let us assume that $\gamma < 0$ what means that the CLC is amplifying. If $|\gamma|$ is sufficiently small the waves emerging from the layer according to (6–9) exist only in presence, at any rate, of one external wave incident at the layer and their amplitudes are determined by the solution of Eqs. (6, 8). Now (see Fig. 8c) $\mathrm{R} + \mathrm{T} > 1$ or $1\text{-R-T} < 0$ what just corresponds to the definition of amplifying medium.

However if the imaginary part of the dielectric tensor, i.e., γ, reaches some critical negative value the quantity $\mathrm{R} + \mathrm{T}$ diverges and the amplitudes of emerging from the layer waves occur to be nonzero even for zero amplitudes of the incident waves. This happens when the determinant of Eq. (6) reaches zero value. At this point, of cause, the amplitudes of emerging waves are not determined by the solution (8) of the linear equations (1) (a nonlinear problem should be solved now). However the points of reducing the determinant of Eq. (6) to zero determine, as we saw above, edge modes (EM) [1,12,13] and the corresponding values of the gain (or negative imaginary part of the dielectric tensor), i.e., the minimum threshold gain at which the lasing happens (see the corresponding discussion for scalar periodic media in [12,13]).

So, the equation determining the threshold gain (γ) at which the lasing happens (zero value of the determinant of Eq. (6) or the denominator of the expressions (9)) occurs to be coinciding with the Eq. (12). However, it should be solved now not relative the frequency but relative to the imaginary part of the dielectric constant (γ).

In a general case, this equation has to be solved numerically. However for a very small negative imaginary part of the dielectric tensor the frequency values of the edge lasing modes are pinned to the frequencies of zero value of reflection coefficient in its frequency beats outside of the stop band edge for the same layer with zero imaginary part of the dielectric tensor [1,8,9]. It is why for this limiting case the threshold values of the gain for the edge lasing modes (or negative imaginary part of the dielectric tensor) may be presented by analytical expressions.

For small $|\gamma|$ and $L|\mathrm{Im}q| \ll 1$ the reflection and transmission coefficients (9) at the frequencies of reflection minima (19) are reduced again to the expressions (21), however with negative γ.

So the R and T may be divergent now and the points of R and T divergence correspond to the lasing at the frequencies of EM and determine the corresponding values of the threshold γ, i.e., a minimum $|\gamma|$ at which the lasing happens for nth reflection minimum:

$$\gamma = -\delta(n\pi)^2/a^3 = -\delta(n\pi)^2/(\delta L\tau/4)^3 \tag{22}$$

As one sees from (22) the threshold values of $|\gamma|$ are inversely proportional to the third power of the layer thickness and a minimal value of $|\gamma|$ corresponds to $n = 1$, i.e., to the edge lasing mode closest to the selective reflection band edge (compare with the analogues corresponding results for the scalar layered media [12,13]). The values of γ given by the Eq. (22) is convenient to use for estimating of the threshold values of γ in the general case and as a zero approximation in the numerical solution of the Eq. (12) for the threshold values. The frequency distance between the consequent edge lasing modes are equal to $\nu_{n+1} - \nu_n = ((\pi/a)^2/2)(2n+1)$, i.e., inversely proportional to the second power of the layer thickness (compare with the corresponding distance between the lasing frequencies in a homogeneous layer which is inversely proportional to the layer thickness).

As it also follows from Figure 7 the different threshold values of γ correspond to the different edge lasing modes (divergent R and T) at Figure 7a–d. It means that one is able to excite separate lasing modes by changing the gain (γ). If the value of γ is between the consequent threshold values of γ for neighboring lasing modes the lasing may not be achieved and the layer may reveal only amplifying properties (see Fig. 8a–c). This means that changing of the pumping wave intensity allows to achieve lasing at the individual edge modes and the lasing intensity is not a monotonic function of the pumping intensity. Because the lasing frequency is determined by the edge mode frequencies there is an option for some variation of the lasing frequency inside the width of the dye line by changing the CLC pitch by means of the temperature variations [5,6] or by application to the layer of external electric or magnetic field [4]. Note, that some times smooth variations of the external agent may result in jump-like variations of the lasing frequency [19] connected with the jumps of the CLC pitch which are sensitive to the surface anchoring [20].

OPTIMIZATION OF PUMPING

The formulas of the preceding sections allow to optimize separately the lasing threshold by reaching coincidence of the lasing frequency with the frequency of first edge lasing mode and to optimize the pumping efficiency by reaching coincidence of the pumping frequency with the frequency of the first maximum in anomalously strong absorption effect [8]. Note, that the same CLC should be simultaneously an absorbing material at the pumping frequency and amplifying one at the lasing frequency. In the present section we shall discuss the possibilities to reach simultaneously the highest efficiency of the pumping and the lowest value of the lasing threshold gain (minimal $|\gamma|$ for negative value of γ at the lasing frequency). The demands of the highest efficiency of the pumping and the lowest value of the lasing threshold gain are contradictory ones for a collinear geometry because they assume that the lasing frequency ω_{l} and the pumping frequency ω_{p} practically coincide with the frequency edges of selective reflection band. However, the lasing frequency ω_{l} is less than the pumping frequency ω_{p}. Note, that this contradiction for the collinear geometry may be overcame by a chance or by a very fine tuning of the lasing parameters if the difference $\omega_{\mathrm{p}} - \omega_{\mathrm{l}}$ is small and ω_{p} coincides with the high frequency edge of the reflection band and ω_{l} coincides with the low frequency edge of the reflection band what results in the following frequencies of the lasing and pumping waves:

$$\omega_p = (\mathrm{c}\tau/2)/[\varepsilon_{0\mathrm{p}}(1-\delta_{\mathrm{p}})]^{1/2}, \quad \omega_l = (\mathrm{c}\tau/2)/[\varepsilon_{0l}(1+\delta_l)]^{1/2},$$
$$\omega_{\mathrm{p}}/\omega_{\mathrm{l}} = [\varepsilon_{0\mathrm{l}}(1+\delta_l)/\varepsilon_{0\mathrm{p}}(1-\delta_{\mathrm{p}})]^{1/2}, \tag{23}$$

where the dielectric constant ε_0 and anisotropy δ are marked by subscripts p and l what relates to the frequency dispersion of the dielectric properties and means that the corresponding parameters have to be taken at the pumping and lasing frequencies, respectively. Note that recently the lowering of the lasing threshold due to the anomalously strong absorption of the pumping wave in the collinear geometry was experimentally observed [11].

Another possibility to reach the lowest threshold in collinear geometry may be realized by applying to the LC material of external electric (or magnetic) field. As it is known [9] in this case due to distortion of the LC helix many diffraction orders exist for light propagating along the helix axes and the pumping and lasing frequencies may be fitted to the frequencies of different diffraction orders. Nevertheless, again the corresponding fitting assumes a very fine tuning of the lasing parameters.

However there is a regular way to reach optimization of the pumping intensity (under assumption that the lasing occurs along the helical axis). One has an option to use a noncollinear pumping without any tuning of the lasing parameters, i.e., the pumping wave propagating at an angle to the helical axis which allows the pumping wave to be at the conditions of the anomalously strong absorption effect. A rough estimate derived from the fact that the lasing and pumping waves experience Bragg scattering gives the following value of the angle θ between the pumping wave propagation direction and the helical axes

$$\theta = \arccos[\omega_{\mathrm{l}}/\omega_{\mathrm{p}}]. \tag{24}$$

To obtain more accurate expression for the pumping wave propagation direction one has to solve the Maxwell equations for light propagating at an angle to the helical axes and find the angle θ corresponding to the conditions of anomalously strong absorption effect [8,9]. Unfortunately, no exact analytical solution of the Maxwell equations is known for this case and there is a need to apply a numerical approach in an exact solving of the Maxwell equations. The full power of the numerical approach demonstrate itself if one takes into account the frequency dispersion of the dielectric constant and of the LC dielectric anisotropy. However these quantities are usually not very well known, so in the experiment, if even the calculated angle θ is known, one, due to the mentioned uncertainties, has to search the actual angle of anomalously strong absorption effect by changing the pumping wave propagation direction.

Under these circumstances a more accurate than (24) approximate expression for θ may be quite useful. The corresponding expression was found [10] in the framework of the dynamical theory of diffraction applied to the case of light propagating at an angle to the helical axes [9,18,21]. The dielectric anisotropy δ plays the role of a small parameter in this theory. Because in many practical cases the dielectric anisotropy δ is quite small ($\delta < 0.1$) the accuracy of the results found in the framework of the dynamical theory may be sufficient to describe the experimental results.

CALCULATION RESULTS

In order to obtain the gain (γ) which corresponds to onset of lasing in CLC layer we investigated the behavior of R (reflection) and T (transmission) coefficients. The divergence of R and T just corresponds to a lasing threshold value of γ. These divergences happen at the point of zero value of the determinant of Eq. (6). This condition gives a direct way of finding EM frequency corresponding to the solution of Eq. (12). For amplifying CLC the solutions of Eq. (12) are presented at Figure 9 for several values of the parameter δL/p (the specific values of γ are also presented at Figure 9 for $\delta = 0.05$, 0.03). Zero values of the determinant of Eq. (6) happen at discrete values of γ and ω. Their values are found for several first EM and the calculations of R and T were also performed to control the solution procedure using the initial value of γ given by the analytic expression (22). The final values of ω_i and γ_i were found by a graphical solution of the problem.

The ω_i values are very close to their values corresponding to zeros of the reflection coefficient R for nonabsorbing CLC determined by (13). The values of γ_i are close to the ones given by the analytic expression (22) only for sufficiently large parameter δL/p.

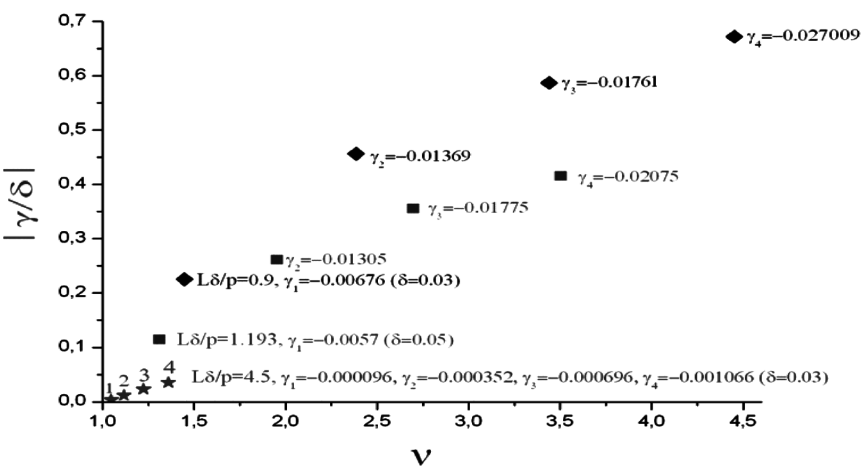


FIGURE 9 Numerical solution of the EM dispersion equation (12) for an amplifying CLC layer for several values of the parameter δL/p shown at the figure (the found values of γ are also given at Figure 9 for $\delta = 0.05$, 0.03).

CONCLUSIONS

The performed analytic and numerical theoretical examination of the edge modes in LC allows to present the physics of these localized modes in a more clear way. As the result, in particular, it happens to be possible to show that there are some real possibilities related to these localized modes for reaching more high efficiency of the DFB lasing in CLC. Namely, possibilities of further reduction of the lasing threshold relative to the one already achieved were predicted [10] and partially experimentally observed [11] as well as possibilities of varying the lasing frequency in this kind of lasing. Some advantage of the CLCs consists in the fact that their parameters are easily variable. So the CLC may be considered as a convenient model object for the studying the lasing in any periodical media, even 3D periodical structures (see paper on lasing in the LC blue phase [22]). Another lucky point from theoretical point of view is availability of exact analytical solution of the Maxwell equations for light propagating along the helical axes. For another periodical media no exact analytical solution is known and one usually applies to the problem the coupled wave approximation [12,13].

What is essential for the experimental observation of the examined anomalously strong absorption effect of the pumping wave it is perfection of the periodical structure sufficient enough to observe beats of the reflection coefficient at the edges of reflection band. Insufficient perfection of the periodical structure leads to lowering of the anomalously strong absorption. A similar lowering of the anomalously strong absorption is connected with a finite frequency width of the pumping wave. The corresponding reduction of the absorption is the result of averaging of the presented above expressions over the frequency width of the pumping wave line [8,9]. A similar influence of the sample perfection on lowering of the threshold lasing gain also takes place. It should be noted also that the accepted above assumption on the absence of dielectric reflection on the boundaries of the CLC layer (the equality of the external dielectric and average dielectric constant of CLC) demands a special experimental care. If in the experiment the mentioned assumption is not met the reflection at the boundaries converts the diffracting polarization into the nondiffracting one what also decreases the anomalously strong absorption and changes the polarization properties of the phenomenon. Another accepted simplification of the problem is connected with the assumption that the absorption in CLC is isotropic. In some cases this assumption may correspond to the real situation. However in the general case the local absorption anisotropy in CLC may be noticeable. So, examining of the problem for the case of anisotropic absorption is quite urgent. And finally, as was already mentioned, because insufficiently precise knowledge of the CLC parameters a practical way to observe the theoretically predicted effects in the experiment is a search of the effect by small variations of the experimental parameters (propagation direction, temperature etc.) around the values calculated according to the previous sections formulas.

In the conclusion note that the results obtained here for spiral media are relevant to any periodic media so the qualitative description of the EM in these media is the same as for CLC and the analytic formula presented above may be used as a some useful guide in the studying of another periodic media. One should also keep in mind that the studied here EMs reveal themselves not only in the lasing but also in another optical phenomena. For example, the nonlinear optical harmonic generation [23] and Cherenkov radiation [24] in a periodic medium is enhanced at the EM frequencies (see also Figs. 29 and 32 in [9] and Figs. 5.10 and 6.2 in [18] related to the nonlinear optical harmonic generation and Cherenkov radiation in CLC).

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